RP-003-001515

RP-003-001515 Seat No. _______

B. Sc. (Sem. V) (CBCS) Examination

February - 2019

BSMT-503-(A) (Theory) : Mathematics

[Discrete Mathematics & Complex Analysis - 1]

(Old Course)

Faculty Code : 003

Subject Code : 001515

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) All the questions are compulsory.

(ii) Numbers written to the right indicate full marks of the question.

1 Answer all the following 20 short answer questions : **20**

(1) Define : Partial order relation.

(2) Totally ordered set.

(3) For the Lattice $(L, \leq)$ and $a, b \in L$ $a \vee (a \wedge b) =$ ______.

(4) Define : Inverse relation.

(5) Define : Irreflexive relation.

(6) Define : Atom in Boolean Algebra.

(7) Define : Minterm.

(8) Define : Boolean function.

(9) Find the atoms of Boolean Algebra $(S_{30}, *, \oplus, ', 0, 1)$.

(10) If $(L, *, \oplus, 0, 1)$ is a bounded lattice then $a * 0 =$ ____.

(11) Define : Analytic function.

(12) State C-R conditions in Cartesian form.

(13) The real part of $f(z)=e^z$ is _____.

(14) If $x+iy=\sqrt{2}+3i$ then $x^2+y=$_____.

(15) The real part of $\frac{2+3i}{3-4i}=$_______.

(16) Define : Closed curve.

(17) If $c: z-z_0=r_0=r_0e^{i\theta}$ then $\int_c \frac{dz}{z-z_0}=$ _______.

(18) If $C:|z|=1$ then $\int_c \frac{dz}{z-2}=$ ____.

(19) Define : Contour.

(20) Define : Jordan Arc.

2 (a) Attempt any three : **6**

(1) Draw Hasse Diagram of (S_{12}, D) where $S_{12}=\{1,2,3,4,6,12\}$.

(2) Explain why $(S=\{2,3\}, D)$ is not a Lattice.

(3) For a Lattice $(L,\leq)$ and $a\in L$ show that $a\vee a=a$ and $a\wedge a=a$.

(4) $\forall a,b\in B$ where B is a Boolean Algebra then show that $a*(a'\oplus b)=a*b$.

(5) State unique representation theorem for Boolean Algebra.

(6) Prove for Boolean expression $(x_1*x_2*x_3)\oplus(x_1*x_2'*x_3)=x_1*x_3$.

(b) Attempt any three : **9**

(1) State and prove modular inequality.

(2) Let $(L, \leq)$ be a lattice then show that $a \wedge b = a \wedge c$ and $a \vee b = a \vee c \Rightarrow b = c$.

(3) Find the cube array presentation of $f(x, y, z) = xy + xz'$.

(4) Prove that sum of all minterms of n variables is 1.

(5) Show that 0 and 1 are unique complements of each other.

(6) Find the minimal sum of product of the following expression.

$$a(x, y, z) = xyz + xyz' + x'yz' + x'y'z$$

(c) Attempt any two : **10**

(1) State and prove Stone's theorem of Boolean Algebra.

(2) Prove that every non-zero element of a finite Boolean Algebra can be expressed uniquely as the sum of atoms of Boolean Algebra.

(3) In usual notations prove that :

$$A(x_1 * x_2) = A(x_1) \cap A(x_2)$$
$$A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$$

(4) Prove that every bounded chain is a distributive Lattice.

(5) Prove that direct product of a Lattice is also a Lattice.

3 (a) Attempt any three : **6**

(1) Show that $f(z) = \overline{z}$ is not analytic.

(2) Prove : $f(z) = \frac{1}{z}$ is analytic.

(3) Find : $\lim_{z \to \infty} \frac{2z - 3}{z - 2i}$.

(4) Find $\int_c \frac{z^2}{z-1} dz$ where $C:|z|=1$.

(5) State Liouville's theorem.

(6) If $C:|z|=1$ find $\int_c \frac{z}{2z-1} dz$.

(b) Attempt any three : **9**

(1) Prove : $u = r^2 \sin 2\theta$ is a harmonic function.

(2) Find an analytic function $f(z)$ such that

$\text{Re}(f'(z)) = 3x^2 - 4y - 3y^2$ and $f(i+1) = 0$.

(3) In usual notations prove that : $\nabla^2 = 4\frac{\partial^2}{\partial z \partial \bar{z}}$.

(4) Evaluate : $\int_c \frac{z^2+3}{z^2(z-4)} dz$ where $C:|z|=1$.

(5) Evaluate : $\int_c \frac{\cosh z}{z^3} dz$ where $C:|z|=1$.

(6) Evaluate : $\int_c \frac{e^{2z}}{(z-1)^4} dz$ where $C:|z|=3$.

(c) Attempt any two : **10**

(1) Obtain Laplace's equation in Polar Form.

(2) State and prove Cauchy inequality.

(3) State and prove Cauchy integral formula.

(4) In usual notations prove that : $\left|\int_c f(z)dz\right| \le ML$.

(5) Find an analytic function $f(z) = u + iv$ such that $u - v = x + y$.